2023 SUPER MOCK ELECTIVE MATHS 2 SOLUTION

ELECTIVE MATHEMATICS MARKING SCHEME

Q1	SOLUTION	*MARKS*
	$2x^2 11x + 9 = 0$	
	$\sigma + \beta = -\frac{11}{2}$,	***B1 for sum of roots***
	$\sigma\beta = \frac{9}{2}$	***B1 for product of roots***
	$(\sigma + \beta)^3 = \sigma^3 + 3\sigma^2\beta + 3\sigma\beta^2 + \beta^3$	***B1***
	$\sigma^3 + \beta^3 = (\sigma + \beta)^3 - 3\sigma\beta(\sigma + \beta)$	***M1*** ***M1 simplification***
	$8(\sigma^3 + \beta^3) = 8\left[\left(\frac{-11}{2}\right)^3 - 3\left(\frac{9}{2}\right)\left(\frac{-11}{2}\right)\right]$ =-737	***A1*** ***6marks***
Q2	$f(x+h) - f(x) = (x+h)^2 + \frac{3}{x+h} - \left(x^2 + \frac{3}{x}\right)$	***M1***
	$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + \frac{3}{x+h} - \left(x^2 + \frac{3}{x}\right)}{h}$	***M1***
	$\frac{x^2 + 2xh + h^2 + \frac{3}{x+h} - x^2 - \frac{3}{x}}{h}$	***M1***
	$\frac{2xh + h^2 + \frac{3h}{3x(x+h)}}{h}$	***M1 for simplification***
	$\lim_{h \to 0}\left(2x + h - \frac{3}{3x(x+h)}\right)$	***M1 for finding limit***
	$\frac{dy}{dx} = 2x - \frac{3}{x^2}$	***A1*** ***6marks***
Q3	$f(x) = 2x^3 + 9x^2 + mx - n$ $x^2 + x - 2 = (x-1)(x+2)$ $x = 1 \; or \; -2$	***M1 for either x = 1 or x=-2***
	$f(1) = 2(1)^3 + 9(1)^2 + m(1) - n$ $-11 = m - n$	***M1***
	$f(-2) = 2(-2)^3 + 9(-2)^2 + m(-2) - n$ $20 = 2m + n$ $from \; m = n - 11$ $20 = 2(n - 11) + n$	***M1***

	$3n = 42$ $n = 14$ $m = 14 - 11, \quad m = 3$	***M1 for solving*** ***A1 for n = 14*** ***A1 for m = 3*** ***6 marks***
Q4	The gradient function of a curve is $\frac{dy}{dx}$ $\frac{dy}{dx} = 2px + 3$ $Y = px^2 + 3x + c$ At (-2, 0) 4p+c =6 ………………. (1) At $(\frac{1}{2}, 0)$ p+4c = -6 ……………... (2) solving eqns (1) and (2) simultaneously p+4(6-4p) = -6 -15p = - 30 ∴ p = 2	***M1*** ***M1 for equ 1*** ***M1 for equ 2*** ***M1 for solving*** ***M1 for simplifying*** ***A1 for p =2*** ***6marks***
Q5	$\cos\theta = \frac{p.q}{\lvert p\rvert\lvert q\rvert}$ $cos\theta = \frac{-4}{5}$ $\frac{-4}{5} = \frac{\binom{4}{3}.\binom{-2}{m}}{(\sqrt{4^2+3^2}) \times \left(\sqrt{(-2)^2+m^2}\right)}$ $\left(5\sqrt{m^2+4}\right)\left(\frac{-4}{5}\right) = 3m - 8$ $16(m^2+4) = 64 + 9m^2 - 48m$ $16m^2 + 64 = 64 + 9m^2 - 48m$ $7m^2 + 48m = 0$ $m(7m + 48) = 0$ $m = -6\frac{6}{7} \ \ and\ m = 0$	***M1 for substitution*** ***M1 for solving*** ***M1*** ***A1 for*** $7m^2 + 48m = 0$ ***M1 for solving*** ***A1*** ***6 marks***
Q6 a	$a = \frac{dv}{dt} = (24 - 6t)m/s^2$	***M1 A1***
b	$s = \int vdt = \int (24t - 3t^2)dt$ $s = 12t^2 - t^3 + c$ When t=0, s=0 then c=0 $s = 12t^2 - t^3$ $when\ t = 3s$ $s = 12(3)^2 - (3)^3 = 81m$	***M1*** ***A1*** ***M1 for solving*** ***A1 for 81m*** ***6marks***

Q7

d	-3	-1	1	0	3	-5	4	-2	$\sum d = -3$
d^2	9	1	1	0	9	25	16	4	$\sum d^2 = 65$

B2 (-0.5 ee) for table

Give assumed mean, A = 6 and n = 8

a) Mean, $\bar{x} = A\frac{\sum d}{n} = 6 + \left(\frac{-3}{8}\right) = 5.63$ — *M1 A1*

b) Variance, $s^2 = \frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2$

$$= \frac{65}{8} - \left(\frac{-3}{8}\right)^2$$ — *M1 A1*

$$= 7.98$$

6marks

Q8

$$P(female) = \frac{2}{3}; \quad P(male) = \frac{1}{3}; \quad n = 5$$

a. $P(x = 3) = {}^5C_3 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^2$ — *M1*

$$= \frac{80}{243} = 0.3292$$ — *A1 for 0.3292*

b. $P(x \geq 4) = {}^5C_4 \left(\frac{2}{4}\right)^4 \left(\frac{1}{3}\right)^1 + {}^5C_5 \left(\frac{2}{5}\right)^5 \left(\frac{1}{3}\right)^0$ — *M1 A1 for correct substitution*

$$\frac{80}{243} + \frac{32}{243}$$

$$\frac{112}{243}$$ — *M1*

$$= 0.4609$$ — *A1 for 0.4609*

6marks

Q9

a. $P = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} T = \begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}$

B1 for $P = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$

B1 for $T = \begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}$

b. $PT = \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -2 & 3 \end{pmatrix}$ — *M1*

$$= \begin{pmatrix} 13 & -3 \\ -5 & 13 \end{pmatrix}$$ — *A1*

c. $\begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ — *M1*

$$= \begin{pmatrix} -6 \\ 12 \end{pmatrix}$$

$$= (-6, \quad 12)$$ — *A1*

d. $Det\ T = 3 \times 3 + 2 \times 1 = 11$ — *M1*

	$$Inverse\ of\ T = \frac{1}{11}\begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix}$$	***M1***
	$$= \begin{pmatrix} \frac{3}{11} & \frac{-1}{11} \\ \frac{2}{11} & \frac{3}{11} \end{pmatrix}$$	***A1***
	e. $\begin{pmatrix} 3 & 1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$	***M1***
	$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{11} & \frac{-1}{11} \\ \frac{2}{11} & \frac{3}{11} \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$	***M1***
	$$= \begin{pmatrix} \frac{1}{11} \\ \frac{8}{11} \end{pmatrix}$$	
	$$= \left(\frac{1}{11}, \frac{8}{11}\right)$$	***A1***
		13marks
Q10	a.	

x	0	1	2	3	4	5
$\frac{1}{\sqrt{x+1}}$	1.000	0.5000	0.4142	0.3660	0.3333	0.3090

B2 for table (-0.5 ee)

	$$\int_0^5 \frac{dx}{\sqrt{x+1}} = \left[\frac{1}{2}(1 + 0.3090) + (0.5 + 0.4142 + 0.3666 + 0.3333)\right]$$	***M1***
	$$= (0.6545 + 1.6135)$$ $$= 2.268$$	***A1***
	b. i) $y = x^3 - 4x^2 + 5x - 2$ $$\frac{dy}{dx} = 3x^2 - 8x + 5$$	***M1***
	When x=3 Gradient at P, $\frac{dy}{dx} = 27 - 24 + 5 = 8$	***A1 for 8***
	ii) x=3, y= 27-36+15-2 =4	***M1*** ***A1 for 4***
	$$y - 4 = 8(x - 3)$$ $$y - 8x + 20 = 0$$	***M1*** ***A1 for*** $y - 8x + 20 = 0$
	iii) $y - 4 = \frac{-1}{8}(x - 3)$ $$8y + x - 35 = 0$$	***M1*** ***A1***
	iv) Gradient= $\frac{dy}{dx} = 3x^2 - 8x + 5 = 5$ $$x(3x - 8) = 0$$	***M1***

	$x = 0 \ or \ 2\,{}^{2}/_{3}$	*A1* *13marks*
Q11	a. *i)* $\cos(\theta + 30^{\circ}) + \cos(\theta - 30^{\circ}) = \cos 30$ $2cos\theta cos30 = cos30$ $2cos\theta = 1$ $cos\theta = \frac{1}{2}$ $\therefore \theta = 60^{o}, 300^{o}$ ii) $cos2\theta - cos\theta = 0$ $cos^2\theta - sin^2\theta - cos\theta = 0$ $2cos^2\theta - 1 - cos\theta = 0$ $let \cos\theta = x$ $2x^2 - x - 1 = 0$ $(2x + 1)(x - 1) = 0$ $x = 1 \ or \ x = \frac{-1}{2}$ When $cos\theta = -\frac{1}{2}$ $\theta = 120^{o}, 240^{o}$ When $cos\theta = 1$ $\theta = 0^{o}, 360^{o}$ $\therefore \theta = 0^{o}, 120^{o}, 240, 360$ b. *i)* $S_n = 8n^2 - 4n$ When n =1,2,3 $S_1 = U_1 = 8 - 4 = 4$ $S_2 = U_2 = 24 - 4 = 20$ $S_3 = U_3 = 60 - 24 = 36$ *ii)* $U_3 - U_2 = 36 - 20 = 16$ $U_2 - U_1 = 20 - 4 = 16$ $\therefore the\ series\ is\ a\ linear\ sequence$ iii) $U_n = 4 + (n - 1)16$ $= 16n - 16 + 4$ $= 16n - 12$	*M1* *M1* *M1* *A1 for all correct answers* *M1* *M1* *M1 for solving* *A1* *A1* *B1 for any two correct* *B1* *M1* *A1* *13marks*

Q12

Marks	freq	x	$u = \frac{x-A}{c}$	u^2	fu	fu^2
	5	52	-3	9	-15	45
	15	57	-2	4	-30	60
	20	62	-1	1	-20	20
	28	67	0	0	0	0
	12	72	1	1	12	12
	9	77	2	4	18	36
	5	2	3	9	15	45
	3	87	4	16	12	48
	2	92	5	25	10	50
	1	97	6	36	6	35
	$\sum f = 100$				$\sum fu = 8$	$\sum fu^2 = 352$

B5 (Accept Alt solution)

Modal mark = 66

M1 for reading
A1 for 66 (±2)

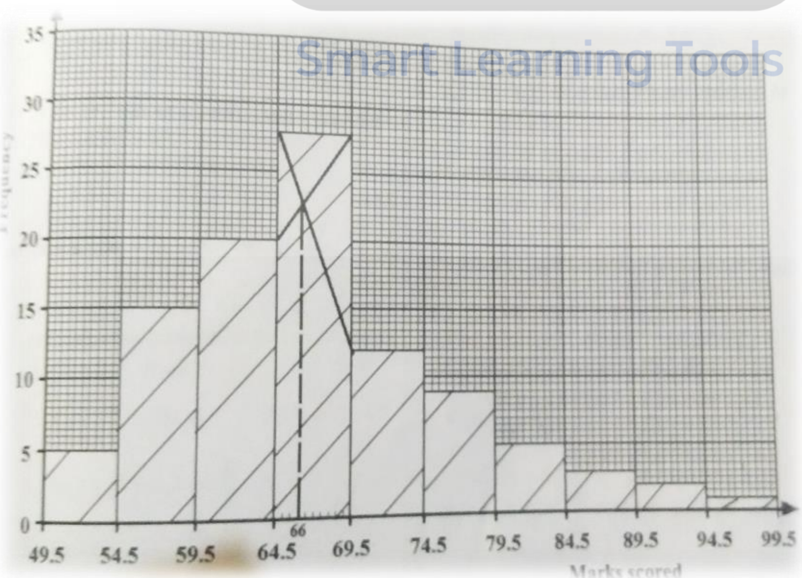
$$mean = A + \left(\frac{\sum fu}{\sum f}\right) \times c$$

$$mean = 67 + \left(\frac{8}{100}\right) \times 5 = 67.4$$

M1 A1
(Accept Alt solution)

$$standard\ deviation, s = 5 \times \sqrt{\left(\frac{352}{100} - \left(\frac{8}{100}\right)^2\right)}$$

$$= 9.4$$



B4 for graph
(-1 ee)

13marks

Q13	a. $\frac{^nC_5}{^nP_4}=\frac{1}{4}$ $\frac{n!}{5!\,(n-5)!}\times\frac{(n-4)!}{n!}=\frac{1}{4}$ $\frac{(n-4)}{5!}=\frac{1}{4}$ $n-4=30$ $n=34$ c. $2\times3\times2\times1=12$ d. $^{15}C_5\times{}^{10}C_5\times{}^{5}C_5$ $=3002\times252\times1=756756\ ways$ e. 2!6!= 2×720=1440 ways	***M1*** ***M1*** ***M1*** ***A1*** ***M1M1A1*** ***M1*** ***M1A1*** ***M1M1A1*** ***13marks***
Q14	a. $a=(i+j)$ $b=(2i+3j)$ $d=(-3i+2j)$ $\lvert a+b\rvert=\sqrt{9+16}=5$ $unit\ vector=\frac{\binom{3}{4}}{\sqrt{9+16}}$ $\frac{3}{5}i+\frac{4}{5}j$ b. $a+b=3i+4j=\binom{3}{4}$ $a+d=-2i+3j=\binom{-2}{3}$ $\lvert a+d\rvert=\sqrt{4+9}=\sqrt{13}$ $5\sqrt{13}cos\theta=\binom{3}{4}\cdot\binom{-2}{3}$ $5\sqrt{13}cos\theta=-6+12$ $cos\theta=\frac{6}{5\sqrt{13}}=0.3328$ c. $\underset{AB}{\rightarrow}=\underset{DC}{\rightarrow}$ $\binom{1}{2}=\binom{x+3}{y-2}$ $x=-2\ and\ y=4$ $\therefore\ position\ vector\ of\ C\ is\ c=-2i+4j$	***M1*** ***M1*** ***A1*** ***M1 A1*** ***M1A1*** ***A1*** ***M1*** ***M1*** ***A1*** ***M1*** ***A1*** ***13marks***

Q15	a. i) $Ft = m(v-u)$ $60 \times 3 = 10(v-5)$ $v = 18 + 5 = 23ms^{-1}$	*M1* *M1A1*
	ii) $F = ma$ $a = \frac{F}{m} = \frac{60}{10} = 6ms^{-2}$	*M1 A1*
	iii) $v = u + at$ $v = 0, u = 23, t = 10s$ $0 = 23 + 10a$ $a = -\frac{23}{10} = -2.3ms^{-2}$ $retardation = 2.3ms^{-2}$	*M1* *M1* *A1 for 2.3ms^{-2}*
	iv) $s = ut + \frac{1}{2}at^2$ $u = 23, \quad t = 10, \quad a = -2.3$ $s = 23 \times 10 - \frac{1}{2} \times 2.3 \times 10^2$ $= 230 - 115$ $= 115m$	*M1* *A1 for 115m*
	b. Let the required force be $F_3 = xi + yj$ $\begin{pmatrix} 3 \\ -4 \end{pmatrix} + \begin{pmatrix} -14 \\ -3 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = -\begin{pmatrix} 3 \\ -4 \end{pmatrix} - \begin{pmatrix} -14 \\ -3 \end{pmatrix}$ $= \begin{pmatrix} 11 \\ 7 \end{pmatrix}$ $\therefore F_3 = 11i + 7j$	*M1* *M1* *A1* *13 marks*